

Balancing building and maintenance costs in growing transport networks

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The costs associated to the length of links impose unavoidable constraints to the growth of natural and artificial transport networks. When future network developments can not be predicted, building and maintenance costs require competing minimization mechanisms, and can not be optimized simultaneously. Hereby, we study the interplay of building and maintenance costs and its impact on the growth of transportation networks through a non-equilibrium model of network growth. We show cost balance is a sufficient ingredient for the emergence of tradeoffs between the network's total length and transport efficiency, of optimal strategies of construction, and of power-law temporal correlations in the growth history of the network. Analysis of empirical ant transport networks in the framework of this model suggests different ant species may adopt similar optimization strategies.

From roads, railways and power grids, to ant trails, leaf veins and blood vessels, transportation structures support the functions necessary to many natural and man-made systems [1–9]. Transport systems are typically represented as spatial networks, where nodes are distinct locations — such as cities or ant nests — and links are physical connections between these locations — such as roads or trails [10]. As transport networks are embedded in a metric space, the length of links is used to quantify the cost of building and maintaining the connections [11]. These costs pose an unavoidable constraint to transport networks, which is intrinsically tied to their spatial nature. Together with the need for efficient transportation and for fault tolerance, costs affect the growth and the topology of transport networks, having profound impact on the systems that rely on them [11].

A great deal of theoretical and empirical research in physics, quantitative geography, and transport engineering has been devoted to understand how diverse constraints influence the evolution of natural and man-made transport networks, and to identify minimal ingredients underlying the emergence of complex topologies [12–17]. The effects of competing design criteria have been explored, such as average shortest path versus link density [18] (or total length [19]), and total length versus synchronizability [20] or centrality [21]. Other models balance the length of newly added links with the gain in centrality [22], or efficiency [4], or analyze the costs and benefits entailed by their creation [23]. However, most of the existing models assume that (i) the network is static and/or constituted by a pre-fixed and known set of nodes, (ii) it is either planned by a central authority, or the result of a completely self-organized process, and (iii) the length of a link is a proxy for both the costs of building *and* maintaining it [11]. Therefore, they neglect that (i) transport systems are typically built iteratively, often lacking information about future developments, as these may be beyond the time horizon of planners [10, 11]; (ii) due to such dynamic evolution, in long-lived infras-

tructures global planning has to compromise with local constraints and competing interests [10], and to alternate with local optimization processes; (iii) building costs and maintenance costs act on different time scales, constituting unavoidable competing constraints that cannot be optimized simultaneously.

These three aspects are strongly related. In a static scenario, the network of minimum length spanning a fixed set of nodes (the *minimum spanning tree*, MST) minimizes both maintenance and building costs [11]. In a dynamic setting, instead, when future node additions are not known in advance, or when the task of building links is partially delegated to local entities, these costs can not be minimized simultaneously. On one side, building cost is minimized by iterating the local rule of “linking each new node to the closest node in the network”. However, the obtained structure (called *dynamical minimum spanning tree*, dMST [22]) does not minimize the total length of the network [17], thus attaining a sub-optimal maintenance cost. On the other side, globally rearranging the network to a MST every time a node is added does minimize the total length, but it requires to destroy old links and rebuild new ones, increasing the building cost. Moreover, maintenance costs must be sustained until links are abandoned or destroyed [24, 25], constraining the network on a longer time scale.

In this paper, we address these open issues by formulating an out-of-equilibrium model for the growth of transport networks in the context where the position of new nodes can not be predicted. By combining the two pure optimization strategies (global, centrally planned MST, and local, decentralized dMST), the model explores the antagonism between the constraints associated with building and maintenance costs.

Model — Our model grows spatial networks starting from a single node and adding one node and one link at a time, so that resulting networks are trees (see Fig. 1). (In real transport networks, fault tolerance is achieved through the presence of cycles [26, 27]. Here we restrict

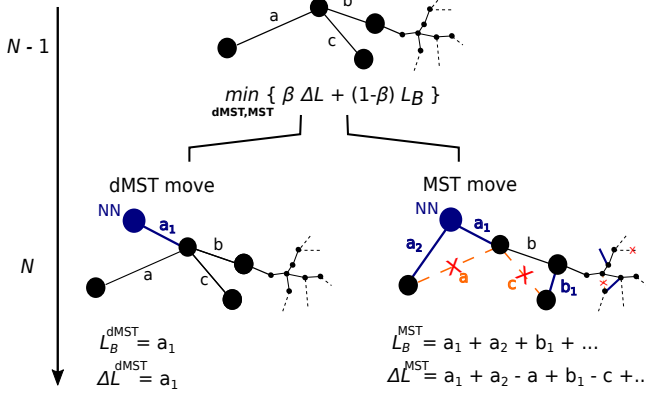


FIG. 1. (Color online) At each time step, the model grows a network by adding a new node (NN) at a random position. Depending on what move minimizes the linear combination of built length L_B and length variation ΔL , NN is either locally connected to the closest node in the network (dMST move) or the network is globally rewired to minimize the total length (MST move). In both cases only one link is added.

to trees for simplicity.) Nodes appear with the flat measure on the unit square. When a new node at position x_N is added to the existing nodes having positions $\{x_0, \dots, x_{N-1}\}$, either it is linked to the closest node (“dMST move”), or a number of links are destroyed and rebuilt in order to obtain the (unique) MST spanning all nodes at positions $\{x_0, \dots, x_N\}$ (“MST move”), such that the functional

$$H(\beta, N) = \beta\Delta L(N) + (1 - \beta)L_B(N) \quad (1)$$

is minimum. L_B is the length that needs to be built, and ΔL is the variation in the total length of the network (these are not equal, as ΔL includes negative contributions from the deleted links). To elaborate, both H^{MST} and H^{dMST} are computed every time a node is added, then the MST move is performed if $H^{\text{MST}} < H^{\text{dMST}}$, and the dMST move otherwise. The “strategy” β is the only parameter of the model, taking values in $[0, 1]$. Setting $\beta = 0$ prioritizes the minimization of L_B (as expected if building costs are dominant), and the network grows only by local dMST moves. Conversely, $\beta = 1$ minimizes ΔL (maintenance costs dominate), and the network is globally rewired to a MST at each step. When the two costs are comparable, intermediate values of β account for both global and local length minimization and the model can alternate between MST and dMST moves. It is useful to express the growth condition $H^{\text{MST}} \geq H^{\text{dMST}}$ in terms of the sum of the lengths of newly built and newly destroyed links, L_B and L_D respectively. For a MST move $\Delta L^{\text{MST}} = L_B^{\text{MST}} - L_D^{\text{MST}}$, while $H^{\text{dMST}} = L_B^{\text{dMST}}$, thus the condition becomes $L_B^{\text{MST}} - \beta L_D^{\text{MST}} \geq L_B^{\text{dMST}}$.

Results — For each value of β from 0 to 1 (by steps of 0.02), we numerically grow 70 networks up to $N_f = 1000$ nodes by the rules of the model. Results are averaged over these 70 networks [Supplemental Material (SM)]. For each

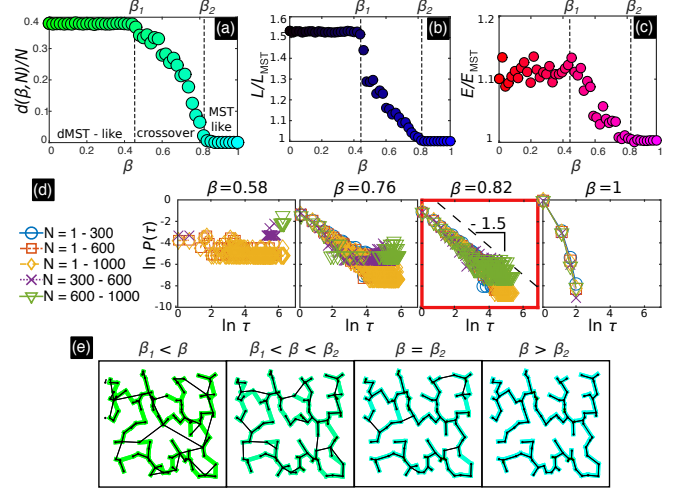


FIG. 2. (Color online) Structural properties and dynamical evolution of networks grown with different strategies β . The normalized (a) Hamming distance, (b) total length, and (c) efficiency reveal three classes of strategies: MST-like, crossover, and dMST-like, separated by two transition points β_1 and β_2 . (d) The probability distribution of the waiting time between two consecutive MST moves $P(\tau)$ for different windows of network size. In β_2 , $P(\tau)$ is a power-law of exponent ≈ 1.5 (highlighted box). (e) Realizations of the model (black thin line) for values of β in the three classes and for $\beta = \beta_2$ on the same 50 nodes sequence (black dots), superposed to the corresponding MST (light bold line).

network, we measure the normalized Hamming distance $d(\beta, N)/N$, defined as the number of links that one has to create (and destroy) in order to turn the network into the MST spanning the same set of nodes, divided by the size of the network N [SM]. This quantity identifies three classes of strategies separated by two transition points, $\beta_1 \approx 0.45$ and $\beta_2 \approx 0.82$ [Fig. 2(a)]. “MST-like” strategies ($\beta > \beta_2$) grow networks at very small Hamming distance from the corresponding MST ($\beta = 1$). “dMST-like” strategies ($\beta < \beta_1$) grow networks similar to the one grown by iterating dMST moves only ($\beta = 0$). “Crossover” strategies ($\beta_1 < \beta < \beta_2$) smoothly interpolate from one extreme to the other. The phase boundaries and the value of $d(\beta, N)/N$ do not depend sensibly on network size after $N \approx 200$ [SM].

The existence of three classes is further confirmed by looking at the total length L and efficiency E of the same networks, normalized by the corresponding MST values and as a function of β [Fig. 2(b) and (c) and SM]. Efficiency quantifies how quickly information and resources are exchanged over a transport network [28, 29], and is often regarded as one of the main design goals in planning and building these networks [8, 12]. It is known that maximizing efficiency competes with minimizing total length [30]. Interestingly, our approach reveals that balancing building and maintenance costs entails a tradeoff between total length and efficiency [Fig. 2(b) and (c)], suggesting that the bias towards efficient transport ob-

served in real networks may emerge under more general conditions, via optimization of a function of length alone.

To better characterize the classes of strategies observed, we introduce the *waiting time* τ , defined as the number of steps from $N = 1$ to the first MST move, and then between two consecutive MST moves. Due to non-stationarity of the process, the probability distribution function $P(\tau)$ of the waiting time depends not only on β , but also on the size of the network [Fig. 2(d) and SM]. Before β_1 , $P(\tau)$ is not defined, as the typical waiting times are larger than those attained by our simulations ($N_f = 1000$). Accordingly, the total length is never minimized through a MST move, and networks in this regime share only a few links with the corresponding MST, typically the shortest ones [Fig. 2(e)]. A mean field estimate of β_1 can be obtained by using the condition for choosing a MST move $L_B^{\text{MST}} - \beta L_D^{\text{MST}} < L_B^{\text{dMST}}$, and assuming that, when β is close to β_1 from above, a MST move destroys and rebuilds nearly all the network's links [Fig. 2(e)]. The typical length of a link in a MST of N nodes can be estimated as the average nearest-neighbor distance among N random points, i.e. $\sqrt{1/cN}$, where c is some constant. Thus, $L_B^{\text{MST}} \sim N\sqrt{1/cN} = \sqrt{N/c}$ and $L_B^{\text{dMST}} \sim \sqrt{1/cN}$, while $L_D^{\text{MST}} \sim \sum_{n=1}^N \sqrt{1/cn} \sim 2\sqrt{N/c}$. The left-hand side of the growth condition becomes $(1-2\beta)\sqrt{N/c}$. Since L_B^{dMST} goes to zero for large N , the condition for at least one MST move to occur in this limit becomes $\beta > 1/2$, which is not far from the observed $\beta_1 \approx 0.45$.

The optimization condition also suggests that, at the onset of the crossover regime, the occurrence of a MST event is tied to the destruction of long links to build short ones. Accordingly, $P(\tau)$ shows MST events are rare and happen typically at large network size [Fig. 2(d)], where the difference between long links, built in the initial steps, and links that would be built in the MST is large. At increasing β , the probability of shorter waiting times increases for small network size, and MST events occur more likely. In the MST-like phase ($\beta \gtrsim \beta_2$), the growth condition is satisfied often, and $P(\tau)$ decays sub-polynomially (exponentially for $\beta = 1$) at all network sizes. Remarkably, the dynamics displays long-range memory at the transition to the minimum-length phase β_2 . Here $P(\tau)$ is a power law of exponent ≈ -1.5 at all sizes, and the waiting time τ has no typical scale (except the cut-off) contrary to the MST-like and dMST-like phases. As a consequence, the occurrence of a MST event is highly unpredictable at β_2 , where waiting times are scale free. (For further discussion see the SM.)

We analyze the performances of different growth strategies in terms of their long-term total cost by means of three time-integrated quantities $\mathcal{L}_B, \mathcal{L}_D, \mathcal{L}_M$, defined as

$$\mathcal{L}_*(N_f, \beta) = \sum_{N=1}^{N_f} L_*(N, \beta) / \sum_{N=1}^{N_f} L_*(N, \beta = 1). \quad (2)$$

These quantities measure how much length was built ($*$ = B), destroyed ($*$ = D), or maintained ($*$ = M) up to

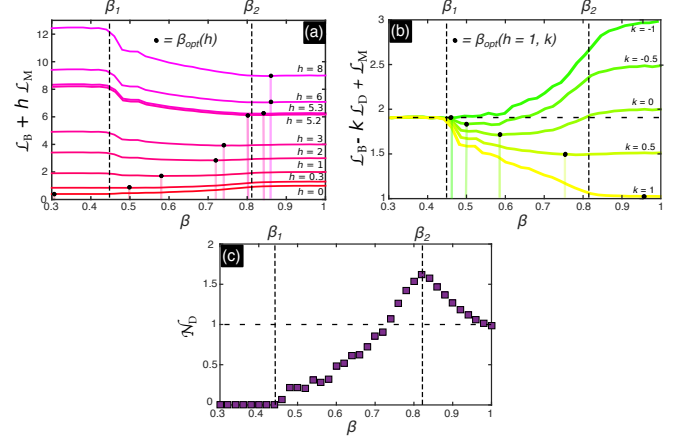


FIG. 3. (Color online) The integrated (up to $N_f = 1000$) cost landscapes (solid lines) for (a) building (\mathcal{L}_B) and maintaining (\mathcal{L}_M), at different values of the ratio h between their unit costs; (b) building, maintaining, and destroying (\mathcal{L}_D) for $h = 1$. $k \in [-1, 1]$ is the unit cost of destroying material (if $k < 0$) or the advantage of recycling (if $k > 0$). The minimum of each cost landscape (dots) is the optimal strategy β_{opt} for the given value of h and k . (c) The integrated number of links that are destroyed and rebuilt (N_D) is maximum in β_2 , revealing high non-extensive costs. Strategies above the horizontal dashed line destroy more links than the pure strategy $\beta = 1$.

$N_f = 1000$ by each strategy β . $L_B(N, \beta)$ and $L_D(N, \beta)$ are the instantaneous lengths built and destroyed between step $N - 1$ and step N , and $L_M(N, \beta)$ is the total length of the network at size N . All the measures are normalized by the values they take in a pure MST dynamics (i.e., at $\beta = 1$) with the same realization of the point process.

In the simple scenario where the costs of maintenance and building per unit length have ratio h , the final cost of a network is given by $\mathcal{L}_B + h \mathcal{L}_M$. Plotting this total cost against β produces a cost landscape for each value of h [Fig. 3(a)]. Each cost landscape has an absolute minimum, which identifies the optimal strategy $\beta_{\text{opt}}(h)$ for the given ratio h . Interestingly, crossover strategies turn out to be optimal for a wide range of values of the ratio h ($0.3 \lesssim h \lesssim 5.2$). More complicated cost scenarios can be analyzed. For example, one may consider that building costs during a MST move may be reduced by recycling the material obtained from the destruction of existing links. On the contrary, when recycling is not possible, disposing of the destroyed material may bear additional costs. Such scenarios can be described by adding the time-integrated destroyed length to the total cost: $\mathcal{L}_B + h \mathcal{L}_M - k \mathcal{L}_D$. The coefficient $k \in [-1, 1]$ is the fraction of destroyed material that can be recycled (if $k > 0$) or bearing additional disposal costs (if $k < 0$). Also in this scenario, crossover strategies play an important role in minimizing the total efforts for construction and maintenance of transport networks [Fig. 3(b), particular case of $h = 1$], realizing nontrivial tradeoffs between the competing costs. The optimal strategy is in the crossover

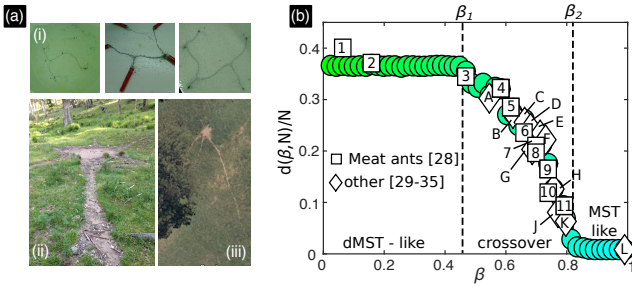


FIG. 4. (Color online) (a) Cost-optimized ant networks: (i) Argentine ants find the shortest path to connect their nests. Credit: Tanya Latty. (ii) Meat ant's nest with departing trails. Credit: Nathan Brown. (iii) Part of a meat ant's transport network from Google Earth. (b) Comparing the Hamming distance of ant transport networks (meat ants [31] are squares, other species [32–38] are diamonds) with our model suggests crossover strategies are relevant for different ant species.

regime even when destroying is as expensive as building and maintaining ($k = -1$), while MST-like strategies are optimal only when recycling strongly lowers the total cost.

All costs considered above are extensive in the length of the transport channels involved. However, length-independent costs may be present in empirical situations, for instance associated to setting up the sites for building and dismantling connections. These “fixed” costs depend on the number of links modified at each step, regardless of their length. We quantify these non-extensive costs via the total number of links that were destroyed (and rebuilt) $\mathcal{N}_D(N, \beta) = \sum_{n=1}^N \mathcal{N}_D(n, \beta) / \sum_{n=1}^N \mathcal{N}_D(n, \beta = 1)$. \mathcal{N}_D is the number of links destroyed at each step, and the sum is normalized by the corresponding MST value, as in (2). Interestingly, β_2 is the strategy requiring the largest number of link deletions, and is therefore a point of strong non-optimality in terms of fixed costs [Fig. 3(c)]. Crossover strategies with $\beta \gtrsim 0.75$ and MST-like strategies require to destroy (and thus to re-build) more links than in the pure MST strategy [Fig. 3(c) horizontal dashed line].

Discussion — Albeit simple, our interpolating model presents a rich behavior, providing a general framework to understand the competing nature of construction and maintenance costs. In doing so, it addresses the interplay of central planning and local growth characterizing the growth of many man-made transport networks, offering insights in the long-term outcome of different short-term construction strategies. Unexpectedly, intermediate growth strategies are optimal in many cost scenarios, as they minimise the long-term total costs entailed by the infrastructure. Moreover, we showed balancing competing costs is a minimal sufficient ingredient for the emergence of the tradeoff between the network's total length and its transport efficiency, which is usually explained by more system-specific principles. Finally, the model displays a transition point with diverging characteristic time, similarly to the phenomenon of critical slowing down close to phase transitions in statistical mechanics, which maxi-

mizes the long-term number of links rewired.

A key premise in the formulation of the model is that the position of new nodes is not known beforehand. If the time scale of the arrival of new nodes is much larger than that of the transport processes on the network itself, then each new node needs to be connected before the position of successive nodes can be taken into account. A possible example in human systems is the evolution of bus routes [39, 40]. New areas can be quickly connected by adding further segments to bus lines stopping nearby (dMST move). However, if the whole network becomes suboptimal in terms of running costs, it may become necessary to re-design it globally (MST move). Our model suggests there may be an optimal re-organization frequency that minimizes the total costs of bus route networks.

In nature, a striking example of cost-constrained transport networks are the trails built by polydomous ant colonies to connect spatially separated nests [3]. Under laboratory conditions, the Argentine ant *Linepithema humile* builds globally optimized transport networks that resemble MST or even Steiner trees (minimum spanning trees where the set of nodes is allowed to be enlarged) [41] [Fig. 4(a), top]. Conversely, the Australian meat ant *Iridomyrmex purpureus* tends to link each newly built nest to the closest one in the colony [17], as in the dMST move in our model [Fig. 4(a), bottom]. For these ants, it has been observed that, during colony growth, suboptimal connections can be progressively substituted with shorter ones and eventually abandoned [42], realising a dynamics similar to the one implemented by our model (although on a time scale comparable to that of node addition). Building on these observations, we used our model as a framework to quantify the trade-off between building and maintenance costs experienced by ants. For 30 published networks constructed by different ant species (*Linepithema humile* [32], *Iridomyrmex purpureus* [31, 33–35], *Formica lugubris* [36, 37] and *Camponotus gigas* [38], see SM for detailed description of the datasets and methods), we measured the normalized Hamming distance from the MST built on the same set of nodes, and assigned a strategy β to each network by comparison with the model prediction for $d(\beta, N)/N$ [Fig. 4(b)]. In all but one of the analyzed trail networks, the rescaled distance $d(\beta, N)/N$ from the corresponding MST departs from 0 at most as much as the typical dMSTs ($d(\beta, N)/N \approx 0.38$). Interestingly, for 27 colonies out of 30, the estimated strategy β is in the crossover regime (as a consequence of the distribution of their rescaled distances). This suggests both maintenance and building costs are relevant in the growth of ant networks, and that different species may share common underlying building principles and optimization strategies. Moreover, the balance between efficiency and total length characterizing most empirical ant networks [30], may be explained by such cost-constrained strategies alone. Alternating between local and global interventions on the network may thus confer evolutionary advantages, and should be taken into account when analysing transport networks.

ACKNOWLEDGEMENTS

We would like to thank D.J.T. Sumpter, J.L. Silverberg, J.L. Denebourg, and M. Cosentino Lagomarsino for constructive feedback and stimulating discussions. A.B. acknowledges funding from the Centre for Interdisciplinary Mathematics (CIM).

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